A Systematic Test of the Independence Axiom

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Introduction

- How does one evaluate: $\frac{1}{3}$ chance of \$20, $\frac{1}{3}$ chance of \$30 and $\frac{1}{3}$ chance of \$10

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- von Neumann Morgenstern: $\frac{1}{3}u(30) + \frac{1}{3}u(20) + \frac{1}{3}u(10)$

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- von Neumann Morgenstern: $\frac{1}{3}u(30) + \frac{1}{3}u(20) + \frac{1}{3}u(10)$
  - Expected Utility Theory
- Used everywhere: Game theory, applied economics
- Advantage: Based on **observables**

Independence Axiom

- $p, q, , r$: Lottery over $\{30, 20, 10\}$
- $\lambda \in [0, 1]$

$$\frac{\begin{array}{ccc} p & \succeq & q \\ & \& & \\ \lambda p + (1-\lambda)r & \succeq & q + (1-\lambda)r \end{array}}{}$$

Independence Axiom

- $p, q, , r$: Lottery over $\{30, 20, 10\}$
- $\lambda \in [0, 1]$

p	$\succeq$	q		p	$\preceq$	q
	&		*or*		&	
$\lambda p + (1-\lambda)r$	$\succeq$	$q + (1-\lambda)r$		$\lambda p + (1-\lambda)r$	$\preceq$	$\lambda q + (1-\lambda)r$

Table: Independence Axiom

Independence Axiom

p	$\succeq$	q		p	$\prec$	q
	&		*or*		&	
$\lambda p + (1-\lambda) r$	$\succeq$	$q + (1-\lambda) r$		$\lambda p + (1-\lambda) r$	$\prec$	$\lambda q + (1-\lambda) r$

Table: Independence Axiom

- If a DM declares p better than q
- Mixing p and q with same lottery r and in same proportion (λ) should not matter.

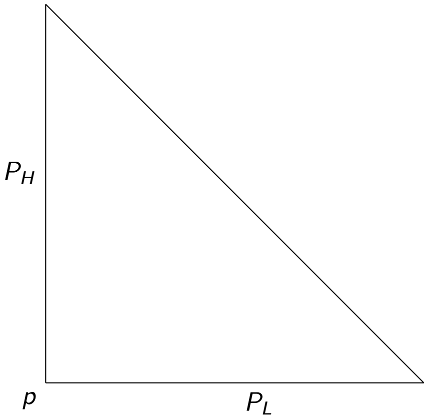
Allais Paradox

- Payoffs: $X = \{4000, 3000, 0\}$
- Lottery l: $(x_1, pr(x_1); x_2, pr(x_2); x_3, pr(x_3))$.
- $p = (4000, 0; 3000, 1; 0, 0)$
- $q = (4000, 0.80; 3000, 0; 0, 0.2)$.
- Mixing Lottery: $r = (4000, 0; 3000, 0; 0, 1)$
- Mixing Probability: $\lambda = \frac{1}{4}$

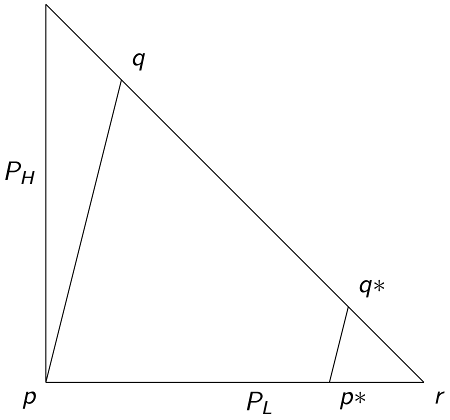
p	$\succ$	q
	&	
$\frac{1}{4}p + (1 - \frac{1}{4})r$	$\prec$	$\frac{1}{4}q + (1 - \frac{1}{4})r$

Table: Allais paradox

Machina-Marshak Triangle



Allais in MM Δ



Certainty Effect

"Consequently, I viewed the principle of independence as incompatible with the **preference for security in the neighborhood of certainty** *shown by every subject... This led me to devise some counter-examples. One of them, formulated in 1952, has become famous as the 'Allais Paradox.' Today, it is as widespread as its real meaning is generally misunderstood." - Maurice Allais (emphasis added)*

- p is better than q: p is certain
- $q*$ is better than $p*$: p is no longer certain
- p looses its certainty appeal

Allais Paradox

- Allais's original intuition was that Independence will be violated "in the neighborhood of certainty," favoring certainty, but **not** otherwise
- Extensive experimental literature:
  - Confirm Allais Paradox: Camerer(1992), Starmer (1992)
  - Suggests otherwise: (Blavatskyy 2010,2013)
- Extensive theory literature: Accommodates certainty effect
  - Dillenberger (2010)
  - Starmer (2003)
- Within "Allais Paradigm"

Allais Paradox: Features

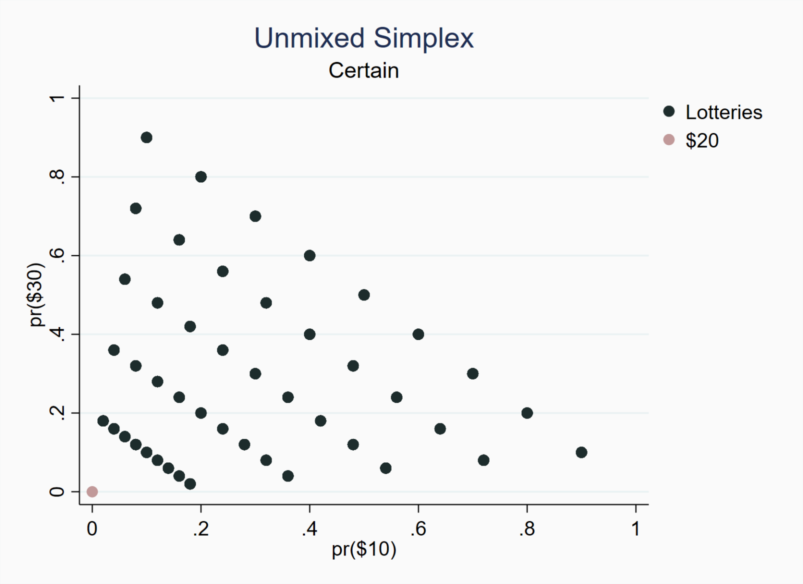
- p: certain
- r: bottom right corner
- High Payoffs

In This Paper...

- Test Independence outside of “Allais Paradigm”
- Test Independence over the entire simplex
- Some decisions involve certainty, some don’t
- Look to see where in the simplex Independence is violated most often
  - Whether these involve certainty or not
  - Can we explain most violations with a preference for certainty?
- Evaluate existing theories
- Empirical regularities for new theories

Design

Questions


Unmixed Simplex
Certain
Lotteries
$20
pr($30)
0
.2
.4
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.8
1
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0
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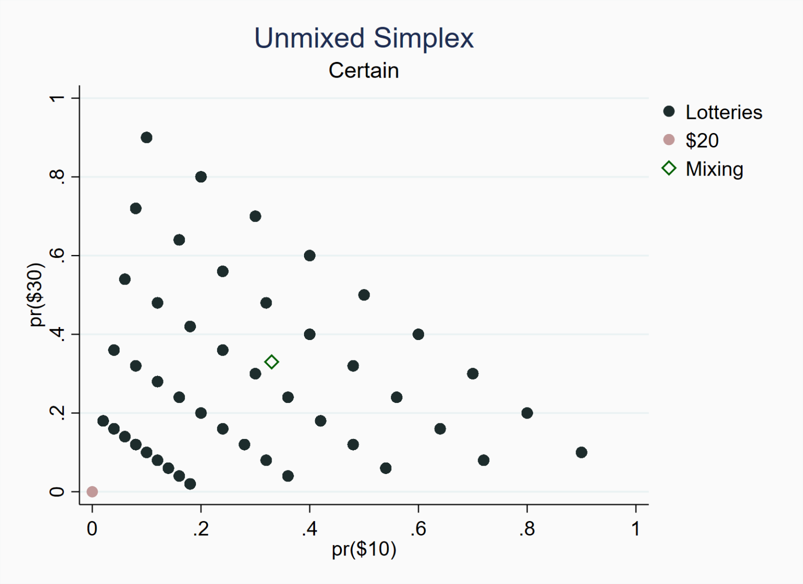
1

Questions



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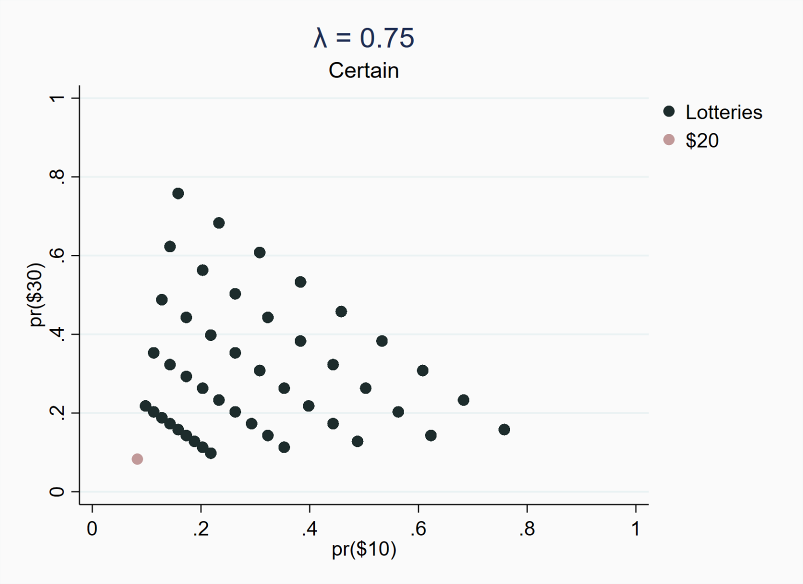

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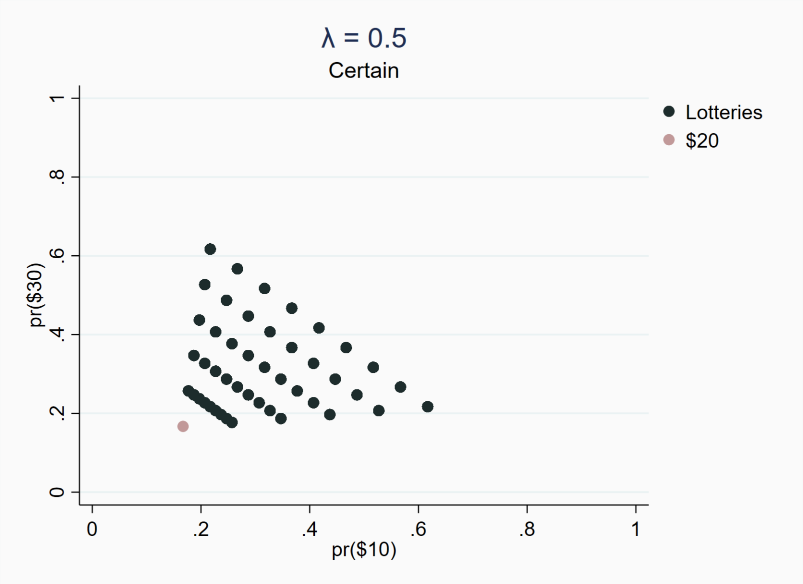

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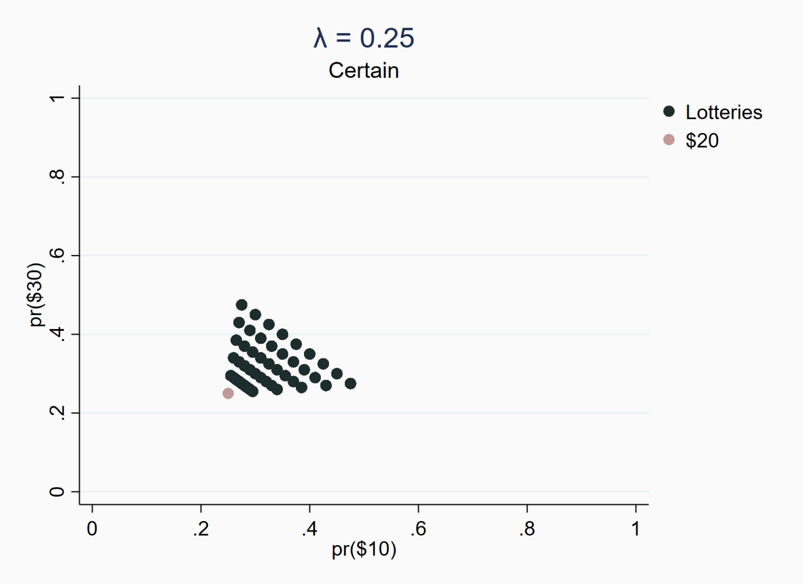
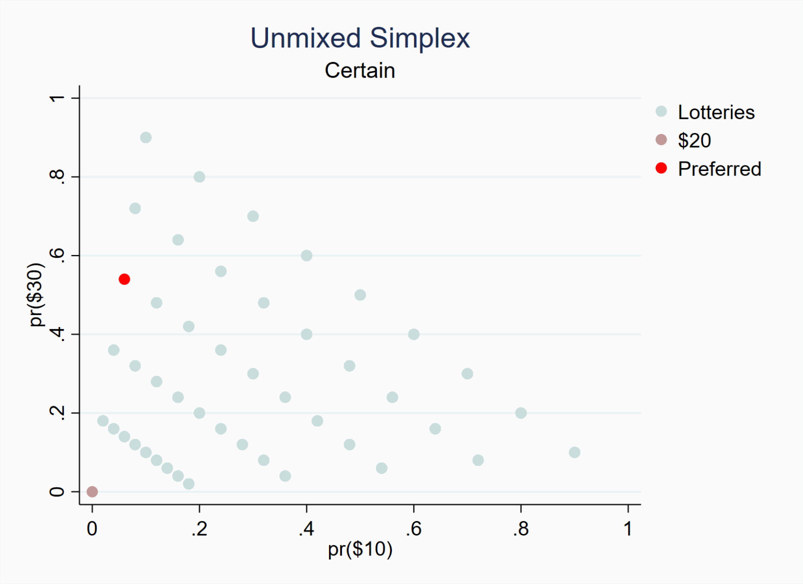


Independence Restriction



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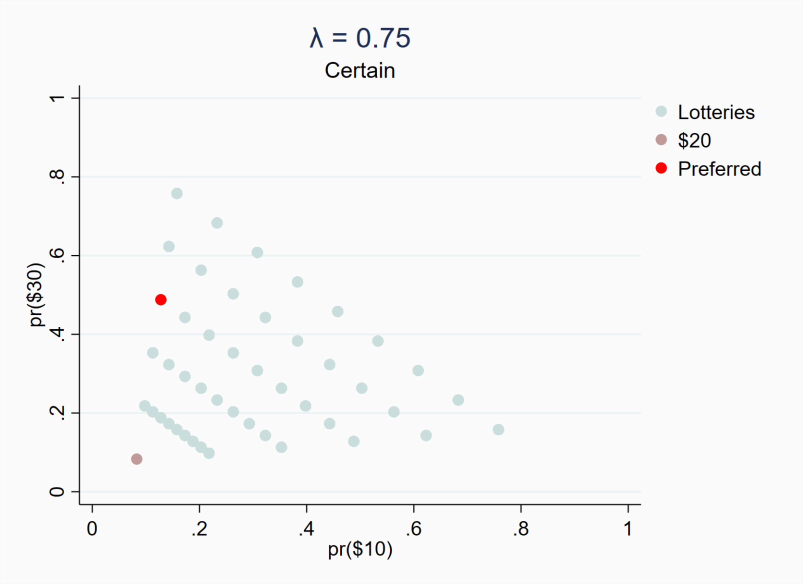

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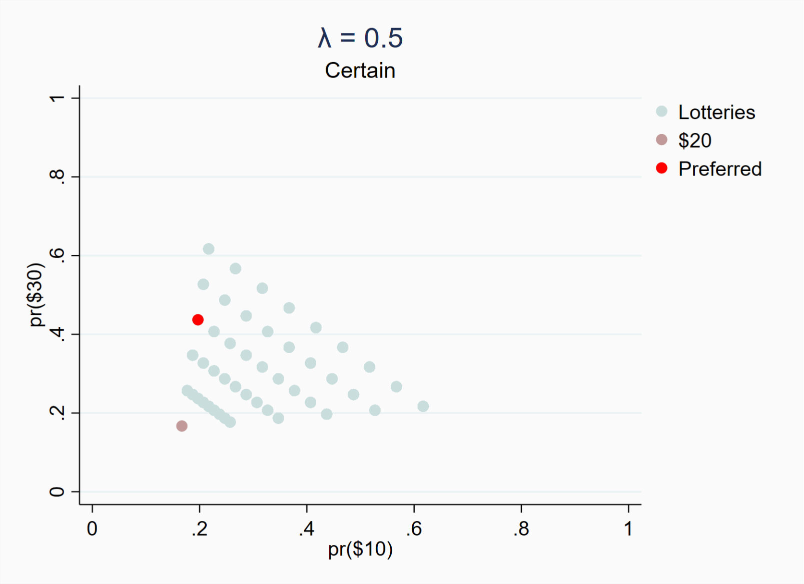

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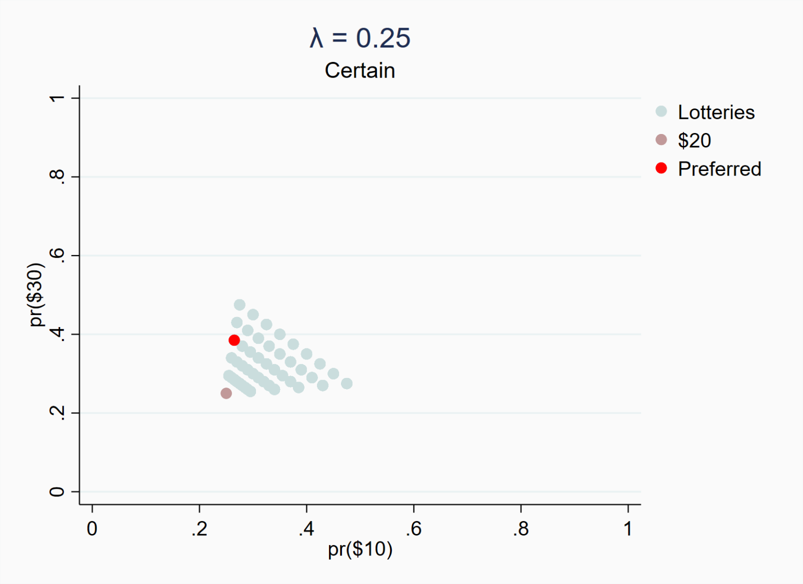
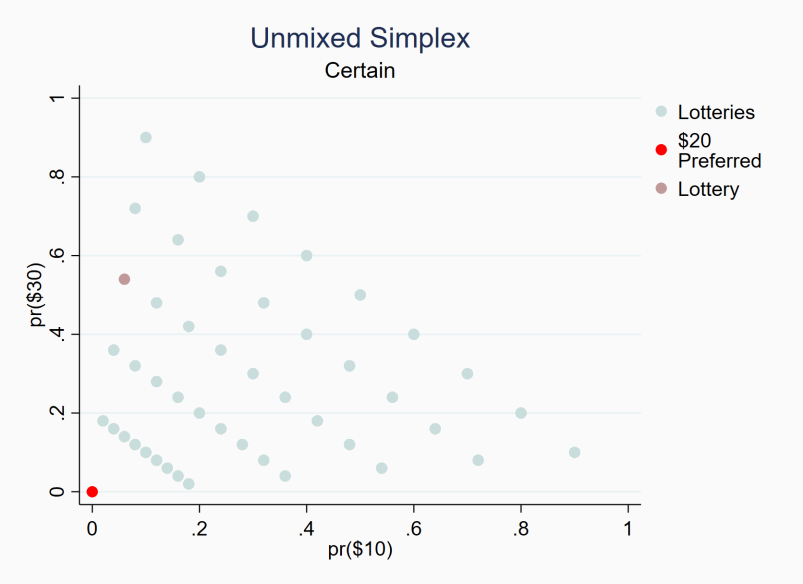


Allais Violation



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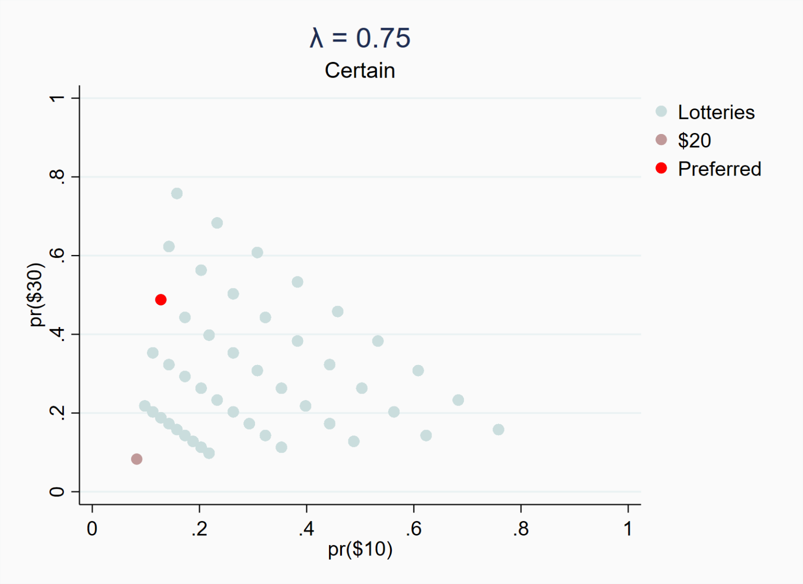

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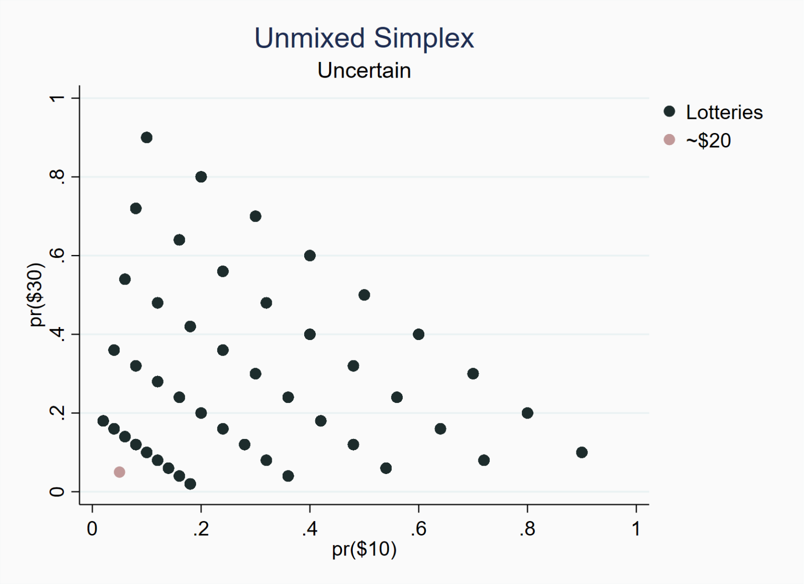

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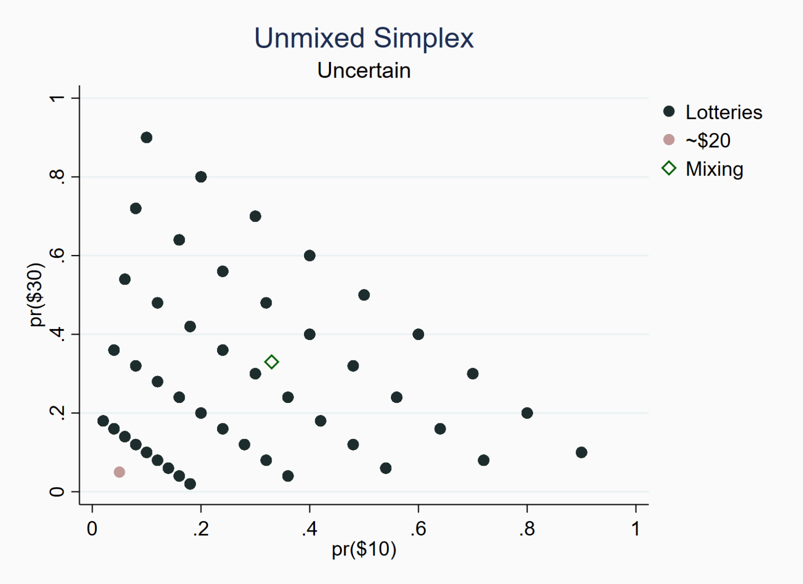
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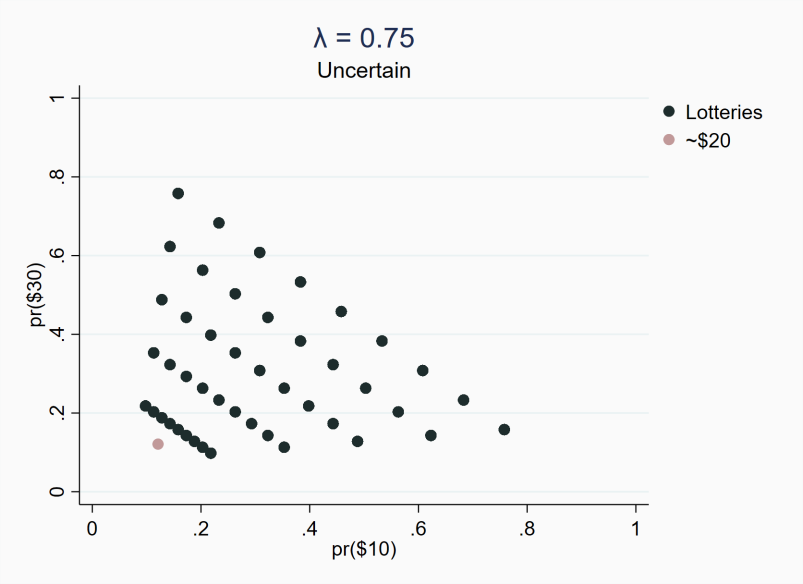

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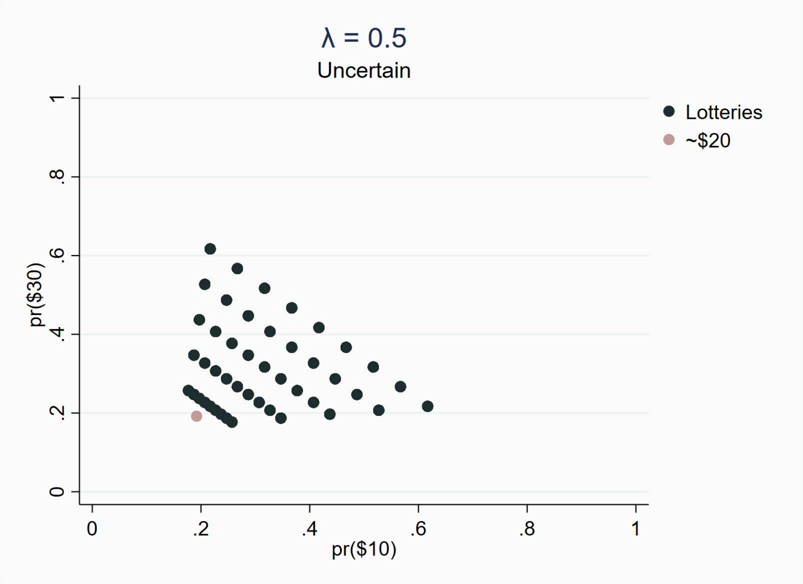

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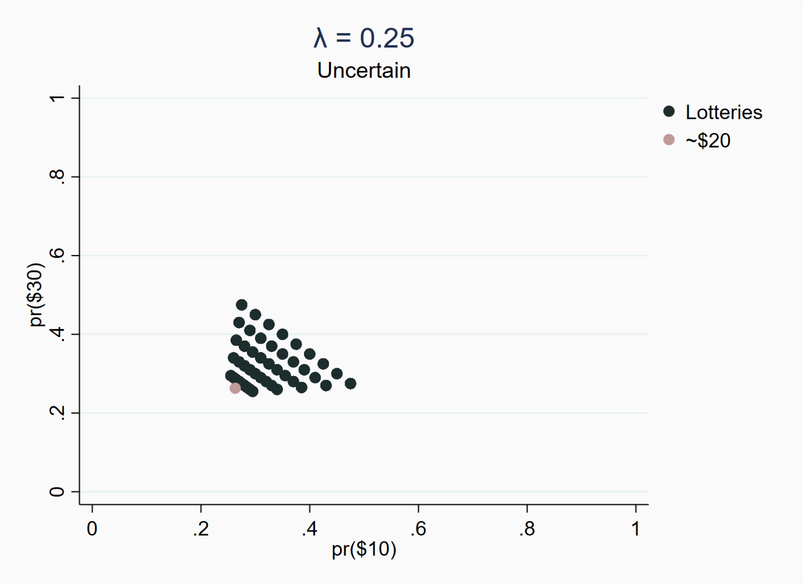
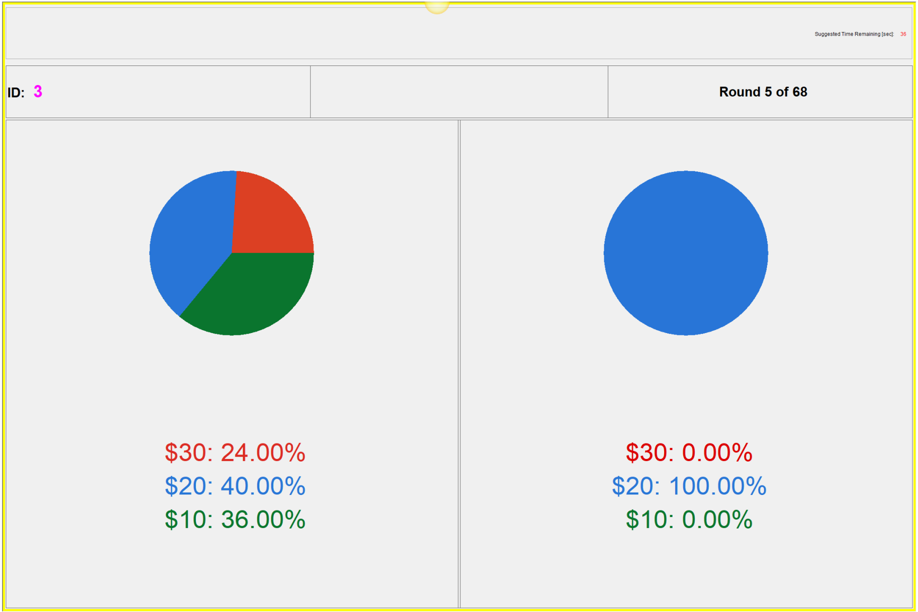


Design

- 360 total questions
  - (45 questions per simplex)$\times$(8 simplices)
- Each subject saw 68 random questions
  - 17 unique $\lambda = 1$ comparison questions
  - Answered $\lambda = \{1, 0.75, 0.5, 0.25\}$ for each
  - 51 tests of independence per subject
  - Randomized independently for certainty and uncertainty
- 147 Subjects
- Average payment: \$20/30 min session
  - "High" stakes

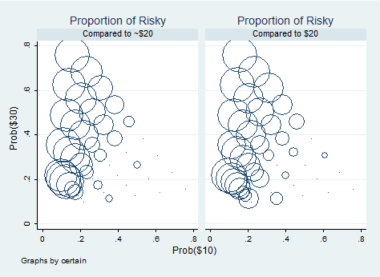

Suggested Time Remaining [sec]: 36
ID: 3
Round 5 of 68
$30: 24.00%
$20: 40.00%
$10: 36.00%
$30: 0.00%
$20: 100.00%
$10: 0.00%

Results

Risk Preference Consistency

Proportion of Risky v/s Safe Choices

$\lambda = .75$



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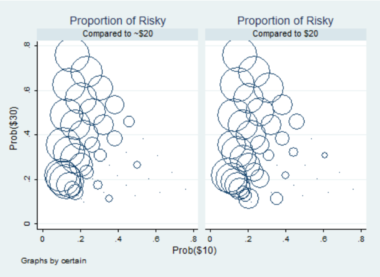
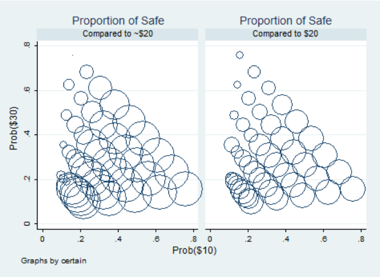
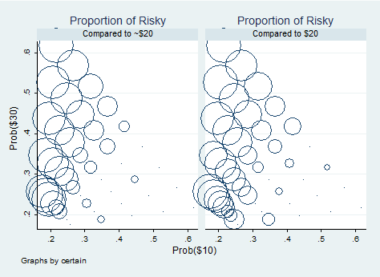




Proportion of Risky v/s Safe Choices

$\lambda = .50$



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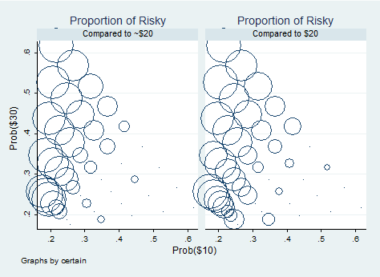

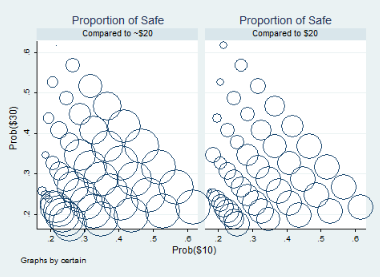

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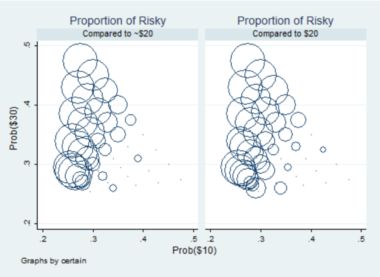
$\lambda = .25$



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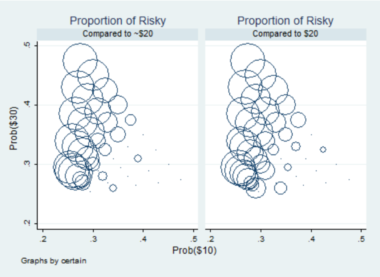
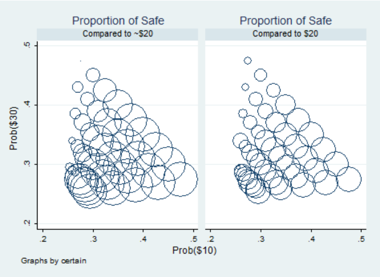




Average Independence Violations

Average Independence Violations

- One pair: λ, certain and p
- Every λ, certain, p:

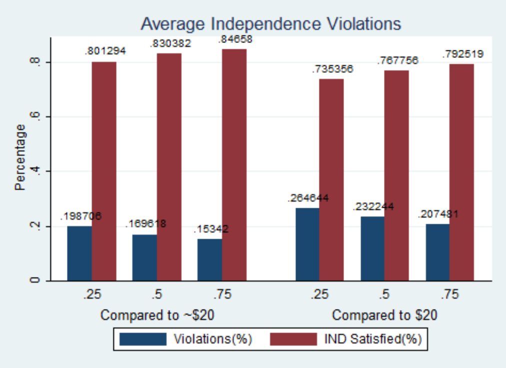
$$Violation(\lambda, certain, p) = \frac{RS + SR}{SS + SR + RS + SS}$$

- Every λ, certain and p:

$$NoViolation(\lambda, certain, p) = \frac{RR + SS}{SS + SR + RS + SS}$$

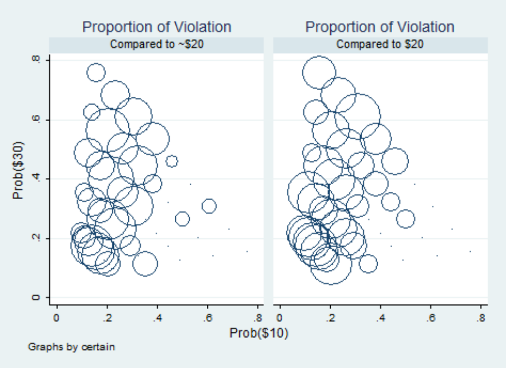
- Average over every pair: λ, certain and p

Aggregate Independence Violations

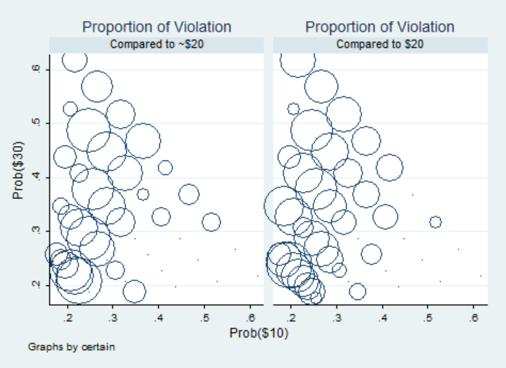


Where are Independence Violations?

Independence Violations: $\lambda = .75$



Independence Violations: $\lambda = .50$



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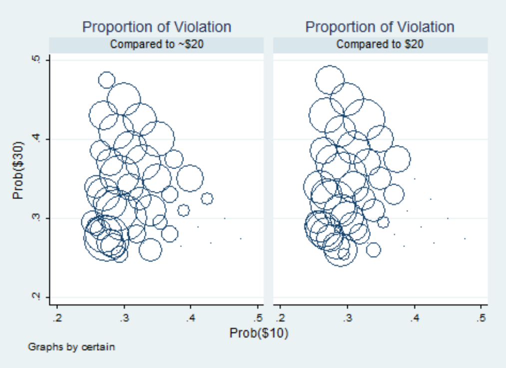
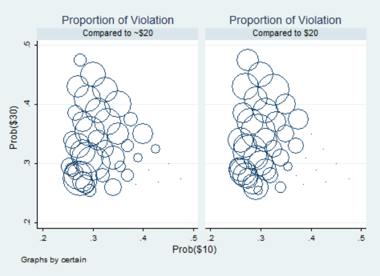
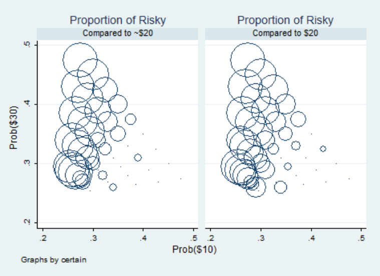


Independence Violations vs Risky: $\lambda = .25$





Pattern of Violations

- Violations correlated with "Risky" region
- Pattern I: Prob(Violation if Risky) $>$ Prob(Violation if Safe)
- Pattern II: Prob(Risky if Violation) $>$ Prob(Safe if Violation)

Violations: Pattern I

- One pair: λ, certain and p
- Every λ, certain, p:

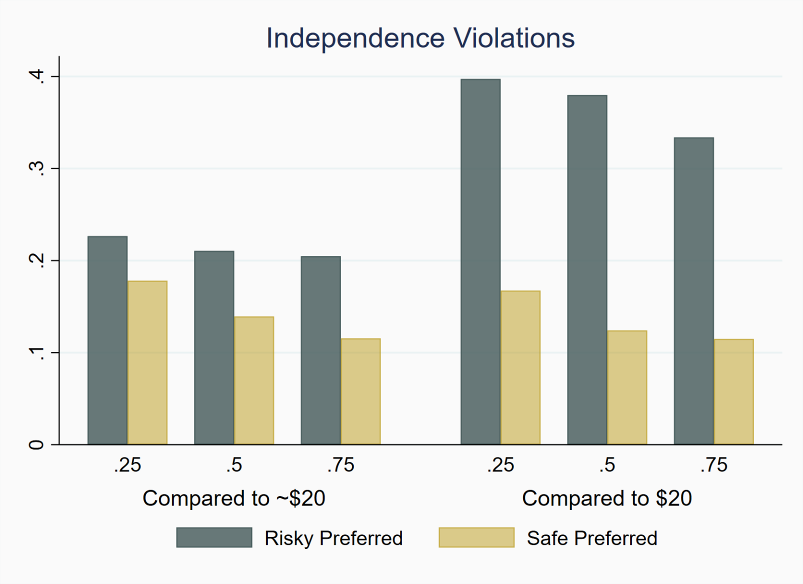
$$SafeViolation(\lambda, certain, p) = \frac{SR}{SR + SS}$$

- Every λ, certain and p:

$$RiskyViolation(\lambda, certain, p) = \frac{RS}{RR + RS}$$

- Average over every pair: λ, certain and p

Independence Violations



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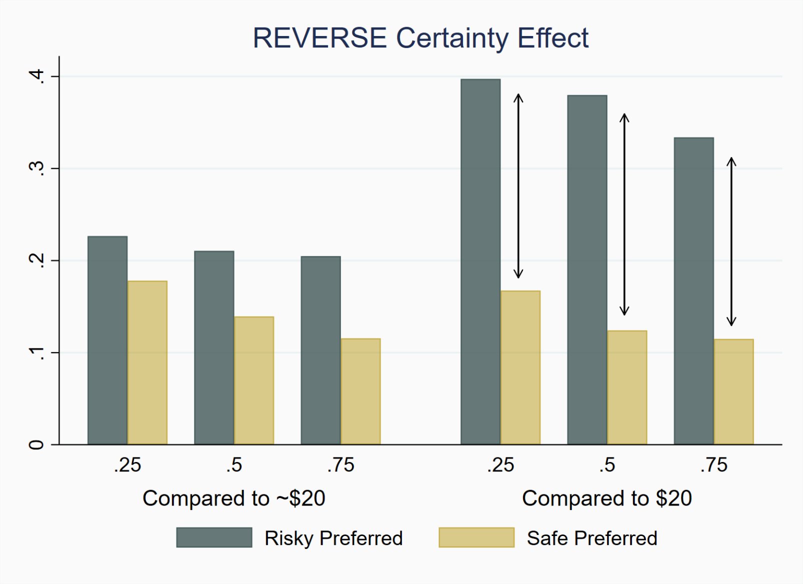

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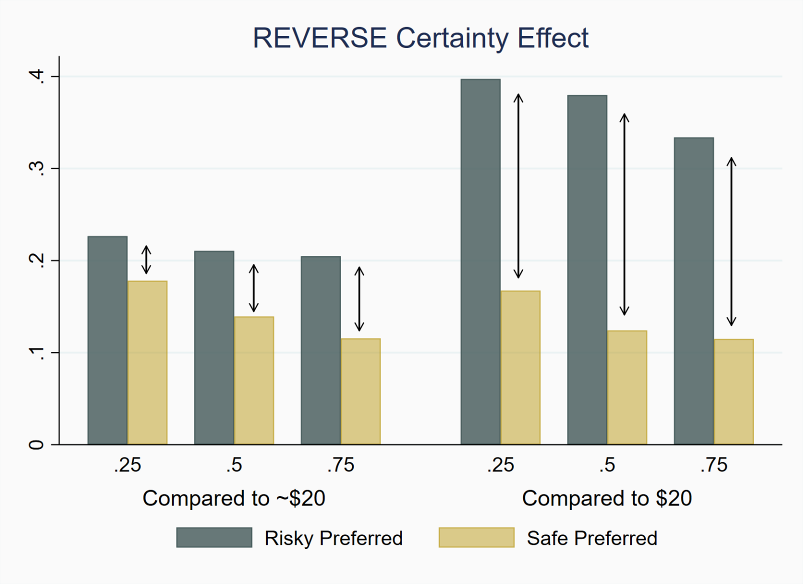

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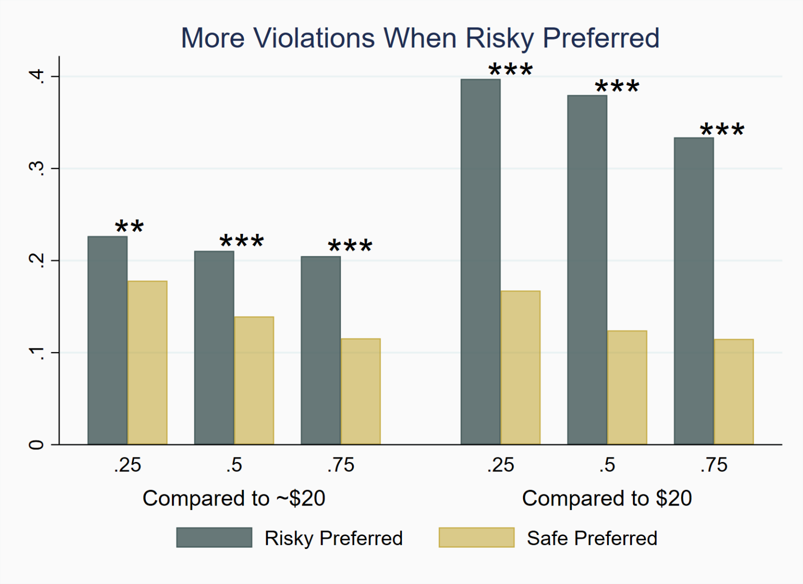

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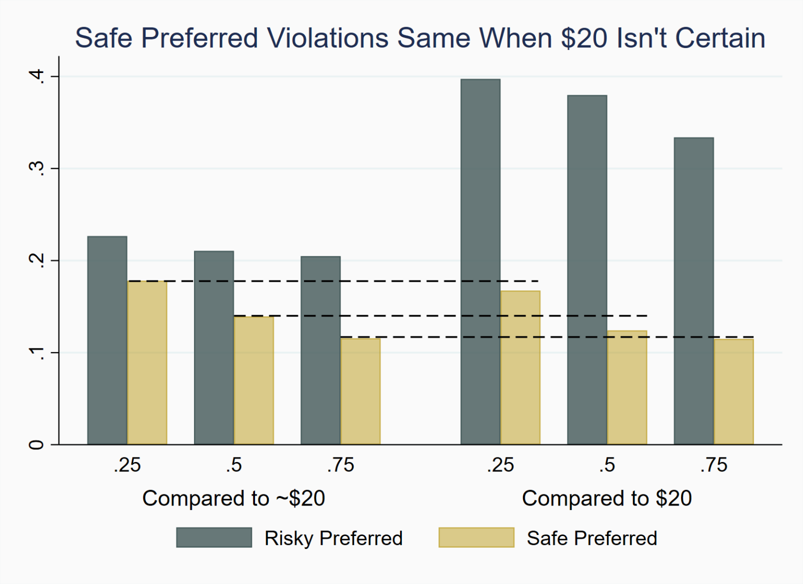


Violations: Pattern II

- One pair: λ, certain and p
- Every λ, certain, p:

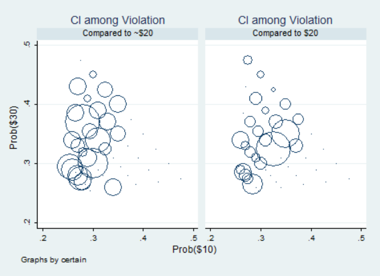
$$SafeViolation(\lambda, certain, p) = \frac{SR}{SR + RS}$$

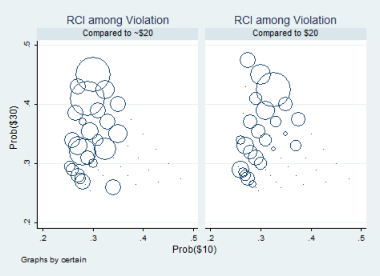
- Every λ, certain and p:

$$RiskyViolation(\lambda, certain, p) = \frac{RS}{SR + RS}$$

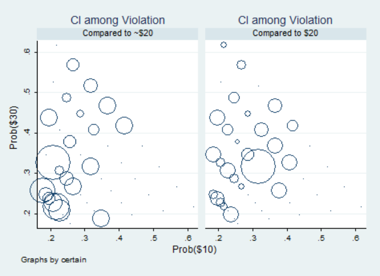
- Average over every pair: λ, certain and p

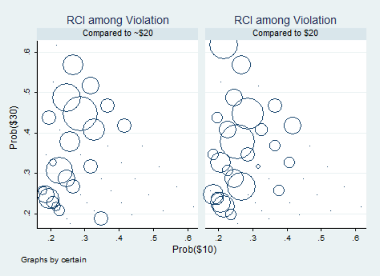
Pattern II Violations: $\lambda = .25$



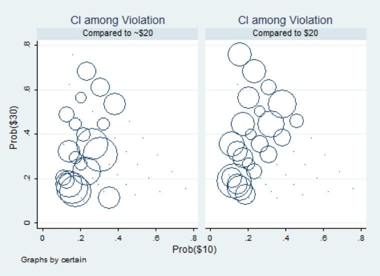


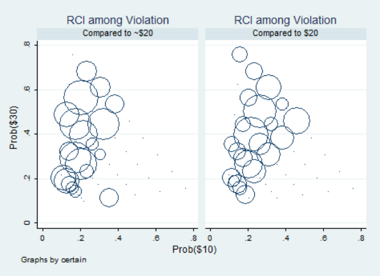
Pattern II Violations: $\lambda = .50$





Pattern II Violations: $\lambda = .75$





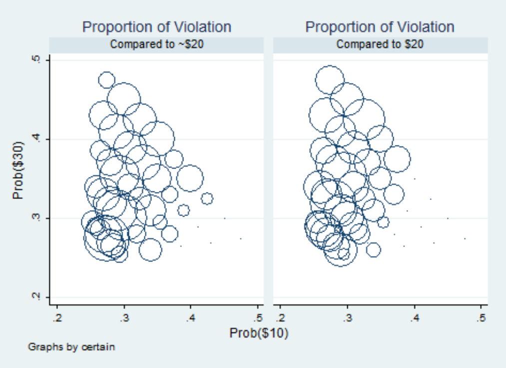
Summary

- We conduct a large-scale test of the independence axiom
- Allow for general violations
- We find the exact opposite of the conventional wisdom
    1. Reverse certainty effect
    2. More violations when the risky lottery is preferred originally
    3. Just as many violations “near” certainty as at certainty itself
- A large literature has studied Allais-type violations
    - In our data, we should be more concerned about the opposite

Future Work

- Collect more data when r is at the bottom right, bottom left and top.

Thank You



Proportion of Violation
Compared to ~$20
Proportion of Violation
Compared to $20
Prob($30)
.2
.3
.4
.5
.2
.3
.4
.5
.2
.3
.4
.5
Prob($10)
Graphs by certain