ASHOKA UNIVERSITY

PhD ECONOMICS ENTRANCE EXAMINATION

Time: 2 hours & 30 mins

Instructions

- Check that this question paper has 16 pages.
- Part I has multiple choice questions. Mark your answers in the OMR sheet. You will be provided only one OMR sheet, so be careful when you make your entries. Each correct answer will earn you 1 point. Each wrong answer will cost you 0.25 points. Your objective should be to maximize your score. There is space available at the back of this question paper for rough work. Please return the OMR sheet and the question paper once you have completed the exam.
- Part II has problem-type questions. Answer any three questions in separately provided answer scripts.
- Please fill in your name, application number, and signature on all answer sheets and at the bottom of this question paper.

Good Luck!

Name: ________________________ Application No.: ________________________

Signature: ____

Part I

Answer all questions.

1. Suppose an urn has b blue balls and r red balls. We randomly pick a ball. If the drawn ball is red, we put two red balls back to the urn. If the drawn ball is blue, we put two blue balls back into the urn. Then we randomly pick a ball again. What is the probability that the second pick is red?

   (a) $\frac{r}{r+b}$

   (b) $\frac{b}{r+b}$

   (c) $\frac{2r}{2r+b}$

   (d) $\frac{2b}{2r+b}$

2. John and Jenny are waiting for Amy at the movie theatre. John says, "If Amy has taken the metro, her probability of reaching on time is 0.7. However if she is driving or has taken a cab, her probability of reaching on time is 0.5 and 0.8, respectively." Jenny adds, "The probability of Amy taking the metro is 0.5, of driving is 0.3 and of traveling by cab is 0.2." Amy is not on time. What is the probability that she did not take a cab?

   (a) $\frac{3}{10}$

   (b) $\frac{17}{50}$

   (c) $\frac{15}{17}$

   (d) $\frac{3}{25}$

3. A random sample of size 8 is drawn from a distribution with probability mass function

$$p(k;\theta) = \theta^k(1-\theta)^{(1-k)}, k = 0, 1; 0 < \theta < 1.$$

The sample values are $1, 0, 1, 1, 0, 1, 1, 0$. The maximum likelihood estimate of θ is

(a) 1
(b) 5/8
(c) 3/8
(d) 0

4. The function $f : \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = |3x| + 4$. For which of the following functions $g : \mathbb{R} \to \mathbb{R}$ does the graph of g intersect the graph of f? (Note that $\mathbb{R}$ denotes the set of real numbers. We use this notation at several places in subsequent questions as well.)

   (a) $g(x) = x - 2$
   (b) $g(x) = 2x - 2$
   (c) $g(x) = 3x + 3$
   (d) $g(x) = 4x - 2$.

5. The function $f : \mathbb{R}_+ \to \mathbb{R}$ is differentiable, strictly concave, strictly increasing and satisfies $f(0) = 0$. Which of the following statements is true about this function? (Note that $\mathbb{R}_+$ denotes the set of non-negative real numbers and f' denotes the first derivative of f)

   (a) $f'(x) < \frac{f(x)}{x}$, for all $x > 0$
   (b) $f'(x) > \frac{f(x)}{x}$, for all $x > 0$
   (c) $f'(x) = \frac{f(x)}{x}$, for some $x > 0$
   (d) None of the above

6. What is the solution to the following optimization problem?

$$\max_{x \geq 0, y \geq 0} \{\alpha y - \beta x\}, \text{ where } \alpha, \beta > 0$$

subject to

$$x + y = 1$$

(a) $x = 1, y = 0$

(b) $x = 0, y = 1$

(c) $x = -1, y = 2$

(d) None of the above

Part II

Answer any 3 of the following questions.

1. Suppose there are two individuals, A and B, who live for two periods, 1 and 2. They get utility from consumption in both periods. There is only one commodity; let's call it money. Let C_1^A and C_2^A be the amounts of consumption of individual A in periods 1 and 2, respectively. A has a utility function defined over his consumption in both periods, and it is given by:
$$\mathcal{U}^A = \sqrt{C_1^A} + \alpha\sqrt{C_2^A},$$
where α is some positive constant less than one, i.e., $0 < \alpha < 1$. Similarly, B has a utility function defined over her consumption in both periods, and it is given by:
$$\mathcal{U}^B = \sqrt{C_1^B} + \beta\sqrt{C_2^B},$$
where C_1^B and C_2^B are B's consumption in periods 1 and 2, respectively, and β is another constant less than one, i.e., $0 < \beta < 1$. The interpretation is following: every period the individuals get utility from consuming some amount, and that utility is given by $u(C) = \sqrt{C}$, where C is that period's consumption. Then, $\mathcal{U}^A$ and $\mathcal{U}^B$ are the individuals' *life-time utility functions* which sum both period's utilities by discounting tomorrow's (i.e., period 2's) utility by a factor. The constant terms, α and β, represent that discounting of future.

   Now suppose the endowments of A and B are given by $\omega^A = (\omega, 0)$, and $\omega^B = (0, \omega)$, respectively, for some $\omega > 0$. Here the first element in the vector corresponds to the individual's period 1 endowment and the second element corresponds to her period 2 endowment. Given their preferences, however, they would prefer to consume positive amounts in both periods. Therefore they might benefit from trading their resources with each other over time. To facilitate this trading across time, suppose a credit market is instituted. Suppose the arrangement is that for every rupee that B borrows from A in period 1, she would have to pay A back $(1 + r)$ rupees in period 2, where r is the interest rate prevailing in the credit market. There is perfect enforcement of these contracts, so that reneging on repayment is not an option.

Suppose $(C_1^{A*},\ C_2^{A*})$, and $(C_1^{B*},\ C_2^{B*})$ are the utility maximizing consumption bundles for A and B, respectively, as functions of ω and r. Now please answer the following questions:

(a) What happens to C_2^{B*} when r goes up?

(b) Now suppose that r^* is the equilibrium interest rate that ensures that the credit market is in equilibrium. What happens to r^* when α goes up?

(c) What happens to r^* when β goes up?

2. Consider an auction where there are two bidders who are bidding for a single object. Each bidder has a private valuation for the object, which is independent of the other's valuation. The bidders are symmetric ex-ante, and each bidder's valuation is independently and identically distributed according to an uniform distribution $U[0, 100]$. The auction rules are as follows: the bidders submit their bids in a sealed envelope, the highest bidder wins the object but both the bidders pay the losing bid. Call this auction Auction A.

(Other auction formats regularly used are:

First Price Auction: where everything else is the same as in the question except for the payment rule. In First Price Auction the highest bidder wins and pays his/her own bid.

Second Price Auction: where everything else is the same as in the question except for the payment rule. In Second Price Auction the highest bidder wins and pays the second highest bid.)

Is the expected revenue of Auction A higher, lower or equal to the expected revenue of a first price auction?

3. Figure 1 plots the effect of fiscal policy in Solow-Swan model.

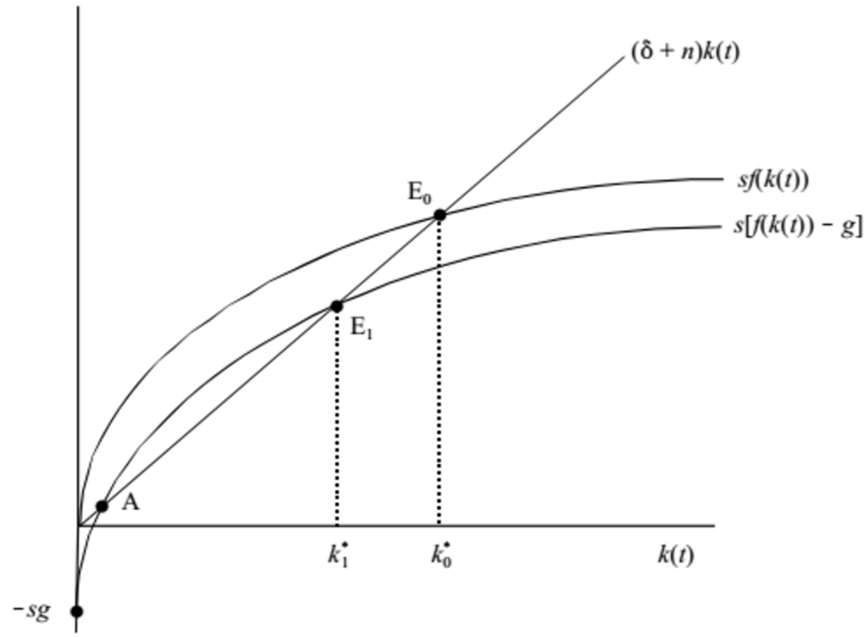

Figure 1: Fiscal policy in Solow-Swan model

Aggregate saving $S(t)$ is proportional to after-tax income, so $S(t) = s[Y(t) - T(t)]$, where $Y(t)$ is national income and $T(t)$ is the lump-sum tax. Under pure tax financing and in the absence of initial government debt, the government budget identity reduces to $T(t) \equiv G$. In per capita terms, we obtain the capital accumulation equation:

$$\dot{k}(t) = s[f(k(t)) - g] - (\delta + n)k(t)$$

Describe the steady state in this model. In the steady state, what is the relationship between the savings rate s, government spending g, and population growth rates on the steady state capital-labour ratio.

4. Consider an economy with identical profit-maximizing firms. A firm produces according to a neoclassical production function with standard properties $F(eL)$, where e is the effort by each employed worker and L is the number of workers the firm employs. The effort exerted by workers employed with the representative firm is an increasing function of the real wage w offered by the firm, the prevailing unemployment rate u, and a decreasing function of the average wage offered by all other firms w_a, i.e. $e(w, w_a, u) = \left(\frac{w - \chi}{\chi}\right)^{\beta}$, where $\chi = (1 - bu)w_a$, $\beta \in (0, 1)$ and $b > 0$. At equilibrium, representative firm pays the prevailing wage, i.e. $w = w_a$. What is the equilibrium unemployment rate?

5. The least squares regression model is given by $\mathbf{y} = \mathbf{X}\beta + \epsilon$ where $\mathbf{X'X}$ is of full rank k and $E[\epsilon^2] = \sigma^2$. The least squares regression coefficient is $\mathbf{b} = [\mathbf{X'X}]^{-\mathbf{1}}\mathbf{X'y}$, and variance estimate is $s^2 = (T - k)^{-1}\mathbf{e'e}$ for $\mathbf{e} = \mathbf{y} - \mathbf{Xb}$. What further assumption do we have to have to make in order for it to be true that $\mathbf{b}$ has minimum variance among the class of unbiased estimators of β?

6. Consider a random sample of 9 observations from a normal distribution $N(0; \sigma)$. The following sample statistics have been obtained from the data: $\bar{X} = -2$, $\sum X_i^2 = 72$. What is the maximum likelihood estimate of the population variance.

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