ASHOKA UNIVERSITY

PhD ECONOMICS ENTRANCE EXAMINATION

May 3, 2020

Time: 2 hours

Instructions

- All questions are multiple choice questions. The question paper has two parts. Part I has 10 questions from statistics and 10 questions from mathematics. Part II has 6 questions from economics.
- Each correct answer in part I will earn you 1 point. Each correct answer in part II will earn you 2 points. Each wrong answer from any of the sections will cost you 0.25 points. Your objective should be to maximise your score.

Good Luck!

Name: _____ Application No.: _____

Signature: _____

Part I

1 Statistics

1. A marketing research firm has collected the following data to determine whether an advertisement campaign has been more successful men or with women.

   - $P(\text{customer is male}) = 0.3$
   - $P(\text{customer saw ad} \mid \text{customer is male}) = 0.4$
   - $P(\text{customer saw ad} \mid \text{customer is female}) = 0.6$

   Determine the probability that a randomly chosen customer who has seen the ad is female.

   (a) $\frac{2}{3}$
   (b) $\frac{3}{4}$
   (c) $\frac{7}{9}$
   (d) $\frac{9}{11}$

2. Suppose the football World Cup had this group: Sweden, Mexico, South Korea and Germany. The situation is as follows: there are only two matches left to be played in the Group: Sweden vs Mexico, and Germany vs South Korea. If Sweden wins its match, Germany must win against South Korea to qualify for the next round. If Sweden loses or draws its match, Germany can qualify with a draw or a win (but not a loss). Suppose all possible outcomes of the two matches are equally likely. If you knew that Sweden lost its game, calculate the probability that Germany qualifies.

   (a) $\frac{1}{6}$
   (b) $\frac{1}{3}$

(c) $\frac{2}{3}$

(d) $\frac{1}{9}$

3. Experience shows that 20% of the people reserving tables at a certain restaurant never show up. If the restaurant has 50 tables and takes 52 reservations, then the probability that it will be able to accommodate everyone is

   (a) $1 - \frac{209}{552}$

   (b) $1 - 14 \times \left(\frac{4}{5}\right)^{52}$

   (c) $\left(\frac{4}{5}\right)^{50}$

   (d) $\left(\frac{1}{5}\right)^{50}$

4. The amount of coffee dispensed into a coffee cup by a coffee machine follows a normal distribution with mean 125 ml and standard deviation 8 ml. Find the probability that one cup is filled above the level of 137 ml. Note that $P(Z \leq 1.5) = \Phi(1.5) = 0.9332$; where $Z \sim N(0, 1)$

   (a) 0.0468

   (b) 0.0586

   (c) 0.0668

   (d) 0.0759

5. A gambler is offered the following game. There are a total of 78 balls out of which some are red and rest are black. The proportion of red balls is given by θ where the gambler does not know what θ is, which means, $\theta \sim U[0, 1]$ (where U stand for the cdf of $Uniform$ distribution). The gambler needs to guess the total number of red balls. What's his best guess?

   (a) 34

   (b) 39

(c) 42

(d) 38

6. There has been a mysterious murder in a mansion. Detective Mr. S has been appointed to the case. Several years of experience of reading detective novels tells him for such a murder mystery in 70% of cases the Butler (B) did it and in rest 30% of cases it's the Gardener (G). The delivery boy is the sole witness and he says it's the Butler. But Mr. S knows in 40% of cases the witness lies. What is the probability that the gardener G is the murderer?

   (a) 40%

   (b) 30%

   (c) 22%

   (d) 36%

7. A six-sided die is rolled thrice. What is the probability that the sum of the rolls is at least 5.

   (a) $\frac{4}{216}$

   (b) $\frac{5}{216}$

   (c) $\frac{211}{216}$

   (d) $\frac{212}{216}$

8. In a market there are a very large number of shopkeepers a fraction λ of whom accept payments through Paytm, while a fraction $1 - \lambda$ are cash only. The true value of λ is not known to the public. There is a very large number of statisticians. Each of these statisticians survey n (n is large) shopkeepers and calculate the value for $\hat{\lambda}$, the proportion of Paytm accepting shopkeepers from their survey data. This means the estimate for $\hat{\lambda}$ vary from statistician to statistician. Which of the following options best approximates the PDF (or PMF) of $\hat{\lambda}$.

(a) PDF: $f(\hat{\lambda}) = \frac{1}{\sqrt{\frac{2\pi\lambda(1-\lambda)}{n}}} e^{-\frac{1}{2}\frac{(\hat{\lambda}-\lambda)^2}{\lambda(1-\lambda)/n}}$

(b) PMF: $Pr(\hat{\lambda} = k/n) = {}^nC_k \lambda^k (1-\lambda)^{n-k}$ for $k = 0, 1, 2, ..., n$

(c) PMF: $Pr(\hat{\lambda} = k/n) = \frac{\lambda^k e^{-\lambda}}{k!}$ for $k = 0, 1, 2, ..., n$

(d) PDF: $f(\hat{\lambda}) = 1$

9. Let Y be a random variable having the density function f given by $f(y) = y/2$ for $0 < y < 2$ and $f(y) = 0$ otherwise. Determine constants a and $b > 0$ such that the random variable $a + bY$ has lower quartile 0 and upper quartile 1.

   (a) $a = 1, b = \sqrt{3}$

   (b) $a = -\frac{3}{2}, b = \frac{3}{2}$

   (c) $a = -\frac{\sqrt{3}+1}{2}, b = \frac{\sqrt{3}+1}{2}$

   (d) Insufficient information

10. An estimator $\bar{\mu}_Y$ of the population value μ_Y is more efficient when compared to another estimator $\hat{\mu}_Y$, if

    (a) $E(\bar{\mu}_Y) > E(\hat{\mu}_Y)$

    (b) $\bar{\mu}_Y$ has a smaller variance.

    (c) The c.d.f. of $\bar{\mu}_Y$ is flatter.

    (d) Both estimators are unbiased, and $var(\bar{\mu}_Y) < var(\hat{\mu}_Y)$

2 Mathematics

1. Let $z = u^7 + 2v^6$, where $u = x^3 + 3$ and $v = x^2 - 1$. Find $\frac{dz}{dx}$.

   (a) $12(x^3+3)^6 + 7(x^2-1)^5$

   (b) $7(x^3+3)^6 + 12(x^2-1)^5$

(c) $21x^2(x^3+3)^6 + 24x(x^2-1)^5$

(d) $24x^2(x^3+3)^6 + 21x(x^2-1)^5$

2. If f is a continuously differentiable real-valued function defined on the open interval $(-2, 2)$ such that $f(0) = -3$ and $f'(x) \leq 5$ for all x, what is the greatest possible value of $f(1)$? (Note that f' denotes the first derivative of f)

   (a) 13

   (b) 1

   (c) 7

   (d) 2

3. The function $f : \mathbb{R} \to \mathbb{R}$ is defined as $f(x) = \frac{x-1}{x-a} + \frac{x+2}{x+a}$. For how many values of a is the function continuous at all values of x?

   (a) None

   (b) One

   (c) Four.

   (d) Two.

4. What is the value of the following limit:

$$\lim_{x \to \infty} \frac{(ax-b)^2}{(a-x)(b-x)}$$

   (a) $\frac{1}{a}$

   (b) $-a$

   (c) $\frac{a-b}{ab}$

   (d) a^2

5. The sum of the eigenvalues of the matrix

$$\begin{pmatrix} -1 & -2 & -1 \\ -2 & 3 & 2 \\ -1 & 2 & -3 \end{pmatrix}$$

is

(a) -3

(b) -1

(c) 3

(d) 1

6. Suppose $f : [-1, 1] \to \mathbb{R}$ is a continuous function such that $f(-x) = -f(x)$ for all $x \in [-1, 1]$. Then $\int_{-1}^{1} f(x)dx$ is equal to

(a) $\frac{1}{2}$.

(b) $-\frac{1}{2}$.

(c) 0.

(d) 2.

7. Consider the following three sets:

$$\mathcal{A} = \{(x, y)|2x + 3y = 15, x \geq 0, y \geq 0\}$$
$$\mathcal{B} = \{(x, y)|2x + 3y \leq 15, x \geq 0, y \geq 0\}$$
$$\mathcal{C} = \left\{(x, y)|2x^2 + 3y^2 \leq 15, x \geq 0, y \geq 0\right\}$$

Which of the following statements is true?

(a) Only $\mathcal{B}$ is convex

(b) Both $\mathcal{B}$ and $\mathcal{C}$ are convex

(c) All three of them are convex

(d) None of them are convex

8. What is
$$\lim_{x \to 0} \frac{\sqrt{1+2x} - \sqrt{1+3x}}{x + 2x^2}, \text{ where } x > 0.$$

   (a) 1

   (b) $-\frac{1}{2}$

   (c) 0

   (d) This limit does not exist

9. Consider the function $f(x) = |x-3| + |x+4|$. What are the number of points at which $f(x)$ is (i) not continuous; (ii) not differentiable?

   (a) (i) 2 ; (ii) 2

   (b) (i) 0 ; (ii) 2

   (c) (i) 2 ; (ii) 0

   (d) (i) 0 ; (ii) 0

10. Consider the function $y = -2x^3 + 3x^2 + 36x - 5$.

    (a) the function is concave for $x > \frac{1}{2}$, convex for $x < \frac{1}{2}$

    (b) the function is always concave

    (c) the function is always convex

    (d) the function is concave for $x < \frac{1}{2}$, convex for $x > \frac{1}{2}$.

Part II

Economics

1. Consider the following utility function of a consumer: $U(x, y) = min\{x, y\}$. The consumer has an income of 300 and the price of x is 1 and the price

of y is both 2. She has a choice of moving to another town where the price of x is 1 and the price of y is 3. She will not get a raise in pay from moving. Although she does not mind moving to the new town, the move is equivalent to a cut in pay of A. She would also not mind moving if she got a raise of B. What are the values of A and B?

(a) $A = 75, B = 100$

(b) $A = 25$ and $B = 25$

(c) $A = 50$ and $B = 66.67$

(d) $A = 100$ and $B = 200$

2. Consider a situation where there are 6 roommates. Each roommate has 4 hours of free time that can be spent cleaning their apartment. All the roommates dislike cleaning but like having a clean apartment. Assume everyone chooses simultaneously how much time to spend cleaning.

   If s_j is the time spent by roommate j in cleaning the apartment, then roommate i's utility is given by

$$v_i(s_1, \cdots, s_6) = \sum_{j=1}^{6} s_j - 2s_i.$$

   Measure social welfare as $\sum_{i=1}^{6} v_i(s_1, s_2, \cdots, s_6)$.

   Denote as s_i^* the effort put in by roommate i in cleaning the room in the Nash equilibrium of this game. Denote as s_i^{**} the effort that roommate i should put into cleaning the room at the Pareto optimum of this game. What are the values of s_i^* and s_i^{**} respectively?

   (a) 4, 0

   (b) 2, 4

   (c) 0, 4

   (d) 0, 2

Answer: (c)

3. Consider an economy with a representative household, a representative firm and a government. The household owns labor (fixed at $\bar{L}$) and capital. It maximizes its lifetime discounted utility subject to its budget constraint. Its lifetime utility function is

$$\int_0^\infty e^{-\rho t} \ln c_t dt$$

where ρ is the discount rate and c_t is per capita consumption of the household at time t. The household is charged a labor income tax at the rate τ. The household spends its after-tax labor income and capital income on consumption of goods and capital accumulation. The government collects tax and purchases goods for its own consumption. The representative firm maximizes its profit where its production function is Cobb Douglas:

$$Y_t = K_t^\alpha L_t^{1-\alpha}, \quad 0 < \alpha < 1$$

where Y_t is output, K_t is capital and L_t is labor. All markets are perfectly competitive. Which of the following statements are true?

   (a) Tax rate affects short run private consumption but does not affect short run capital stock or steady state private consumption or capital stock.

   (b) Tax rate affects short run but not long run values of private consumption and capital stock.

   (c) Tax rate affects short run levels of capital stock but does not affect short run private consumption. Steady state private consumption and capital stock are affected by tax rates.

   (d) None of the above.

4. Consider a model where time is discrete and infinite, and agents live for two periods. Therefore in any period t there are two types of agents:

those born in that period called the young and those who were born in period $t-1$ called the old. The lifetime utility function of an agent born in period t is:

$$U(C_t^Y, C_{t+1}^O) = \frac{\left(C_t^Y\right)^\sigma}{\sigma} + \beta \frac{\left(C_{t+1}^O\right)^\sigma}{\sigma}, \sigma < 1$$

where C_t^Y is the consumption in youth and C_{t+1}^O in when they are old (the next period). When young, an agent born in period t earns a wage income W_t. There exists a pension system so that the agent's budget equations are given by:

$$\begin{aligned} C_t^Y + S_t &= W_t - T, \\ C_{t+1}^O &= (1 + r_{t+1})S_t + Z_{t+1}, \\ L_t &= (1 + n)L_{t-1}, \\ L_{t-1}Z_t &= L_t T \end{aligned}$$

where T is the (exogenous) pension contribution paid during youth, S_t is private savings at time t, and Z_{t+1} are transfers received by old at $t+1$. L_t and L_{t-1} are the populations of young and old in period t, and n is the population growth rate ($n > 0$). We assume that the interest rate r_{t+1} is greater than the population growth rate n for all time periods. Which of the following statements are true?

(a) The pension plan makes agents poorer than they would be without such a plan. For a given interest rate and wage rate, the pension plan would increase old age consumption but reduce consumption at young.

(b) The pension plan makes agents richer than they would be without such a plan. For a given interest rate and wage rate, the pension plan increases consumption in both periods.

(c) The pension plan makes agents poorer than they would be without such a plan. For a given interest rate and wage rate, the pension

plan would reduce consumption in both young and old age.

(d) None of the above.

5. Consider the liner regression model $Y = \beta_0 + \beta_1 X + u$ where u is a zero mean random variable that is uncorrelated with the explanatory variable. But since the researcher is unable to observe either the dependent or explanatory variable, the researcher regresses a variable V on a constant and a variable Z where it is known (a) $V = Y + \eta$ and $Z = X + \epsilon$, (b) η and ϵ are zero mean random variables uncorrelated with X.

   Denote the variances as $Var(u) = \sigma_u^2$, $Var(\eta) = \sigma_\eta^2$, $Var(\epsilon = \sigma_\epsilon^2$ and $Var(X) = \sigma_x^2$. Also denote the covariance between η and ϵ as ρ.

   As the sample size increases to infinity, the estimated coefficient on Z in the above regression converges to

   (a) $\beta - \frac{\sigma_\epsilon^2 + \rho\sigma_\eta^2}{\sigma_x^2 + \sigma_\epsilon^2}$

   (b) $\frac{\beta\sigma_x^2 - \rho}{\sigma_x^2 + \sigma_\epsilon^2}$

   (c) $\frac{\beta\sigma_x^2 + \sigma_u^2 - \rho}{\sigma_x^2 + \sigma_\epsilon^2}$

   (d) $\beta - \frac{\sigma_\epsilon^2 + \rho\sigma_\eta^2}{\sigma_x^2 + \sigma_\epsilon^2 + \sigma_u^2}$

6. Consider the model, $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \epsilon$, where $E(\epsilon|x_1) = E(\epsilon|x_2) = E(\epsilon|x_1, x_2) = 0, E(\epsilon^2|x_1) = E(\epsilon^2|x_2) = E(\epsilon^2|x_1, x_2) = \sigma^2$. Suppose a researcher collects data on the relevant variables and estimates the model by ordinary least squares. Call these as Model 1 estimates. Later a very wise professor points out that β_2 is actually zero. So the researcher restimates the model excluding x_2. Which of the following is true?

   (a) Model 1 estimates are biased but their variances is lower than that of estimates in Model 2.

   (b) Model 1 estimates are biased and have variances higher than estimates of Model 2.

(c) Model 1 estimates are unbiased and have variances lower than that of estimates in Model 2

(d) Model 1 estimates are unbiased but have higher variance than that of estimates in Model 2.